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Incremental meshfree approximation of real geographic data

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Abstract Approximation of scattered data is often task in many engineering problems. The radial basis function (RBF) approximation is appropriate for big scattered dataset in n-dimensional space. It is non-separable approximation, as it is based on the distance between two points. This method leads to the solution of an overdetermined linear system of equations.

In this paper, we propose a new incremental approach for meshless RBF approximation which respects the features of the given geographic data such as ridges, peaks, valleys etc., and thereby, the improving approximation of the underlying data is achieved. Moreover, the proposed approach leads to a significant compression of the given dataset and the analytical description of the data is obtained. In comparison with other existing methods, the proposed approach achieves the better results due to respecting the features of the given data.

Keywords: RBF approximation, Stationary points, Extrema, Incremental algorithm, TPS, Point clouds

1 Introduction

Interpolation and approximation are the most frequent operations used in computational techniques. Several techniques have been developed for data interpolation or approximation, but they usually require an ordered dataset, e.g. rectangular mesh, structured mesh, unstructured mesh, etc. Interesting techniques are based on the Radial Basis Function (RBF) method, which was originally introduced by [1], [2]. RBF techniques are widely used across many fields solving technical and non-technical problems, e.g. surface reconstruction [3], [4], [5], data visualization [6] and pattern recognition. It is an effective tool for solving partial differential equations [7], [8]. The RBF techniques are really meshless and are based on collocation in a set of scattered nodes. These methods are independent with respect to the dimension of the space. The RBF techniques express the given

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data using analytical description. Moreover, RBF approximation allows to attain significant compression of the data.

The good placement of the reference points for RBF approximation plays a significant role in terms of the quality of approximation and the compression ratio. For geographic data, the mentioned requirement is fulfilled for placement along features such as ridges, peaks, valleys etc. A new incremental approach for RBF approximation that puts the emphasis on good placement of reference points and significantly improves the compression ratio will be described in this paper.

In the following sections, the fundamental theoretical background needed for description of the proposed approach will be mentioned. The proposed incremental RBF approximation will be described in Sect. 3. In Sect. 4, the results of our proposed algorithm will be presented. Finally, a final discussion of results will be performed.

2 Theoretical Background

In this section, some theoretical aspects needed for description of the proposed incremental approach will be introduced.

2.1 RBF Approximation

For scattered data processing, the RBF approximation can be used. This technique is based on computing the distance between two points and leads to a solution of linear system of equations which can be solved by singular value decomposition, QR decomposition etc. The RBF approximation is described in [9] or [4] in detail.

2.2 Determination of Stationary Points

Stationary points of an explicit function $f(\boldsymbol{x})$ are points where the gradient of the function $f(\boldsymbol{x})$ is zero vector, i.e. all partial derivatives are zero. In the case, when an analytical explicit expression is not known for the given dataset, the piecewise approach [10] based on RBF interpolation can be used for determination of stationary points in the given dataset.

3 Proposed Approach

In this section, the proposed incremental approach for approximation geographic data using radial basis functions is described.

The main influence on the quality of approximation and the compression ratio has a good placement of the reference points. In case of geographic data, placement along features such as break lines leads to better results. Therefore, in the first level, the set of stationary points obtained for the filtered data using algorithm in [10] is used as set of reference points. The filtered data are determined

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by applying a Gaussian low-pass filter to the given dataset. The main reason for filtration of data is a elimination of insignificant stationary points. Moreover, the set of reference points is extended by corners of dataset bounding box due to avoiding problems on the boundary. Now, the RBF approximation (described in Sect. 2.1) is computed and residues $\boldsymbol{r}_1$ are determined. For this purpose, the following equation is used:

$$\boldsymbol{r}_k = |\boldsymbol{h} - f_k(\boldsymbol{X})| \qquad k = 1, \dots, L, \tag{1}$$

where $\{\boldsymbol{X}, \boldsymbol{h}\} = \{\boldsymbol{x}_i, h_i\}_1^N$ represents the given dataset, $f_k(\boldsymbol{x})$ is approximating function in the k^{th} level and L is number of levels.

In every following level $k > 2$, the residues $\boldsymbol{r}_{k-1}$ are filtered by applying the Gaussian low-pass filter due to eliminating insignificant local maxima. Then, the set of stationary points for filtered residues are determined using algorithm in [10] and only local maxima are added to the set of reference points. Moreover, the uniqueness of the added reference points is checked. When the new set of reference points is obtained, the RBF approximation (described in Sect. 2.1) is again computed and residues $\boldsymbol{r}_k$ are calculated using equation (1). The whole process is repeated until the required accuracy of approximation is achieved or the maximum permissible compression ratio is exceeded.

Finally, it should be noted that the value of standard deviation σ_k of Gaussian low-pass filter in k^{th} level is set as:

$$\sigma_k = \begin{cases} \sigma & k = 1, 2 \\ \dfrac{\sigma_{k-1}}{2} & k = 3, \dots, L \end{cases} \tag{2}$$

where σ is initial value. The whole pseudocode is in Algorithm 1.

Algorithm 1: The incremental RBF approximation of geographic data

Input: given dataset $\{\boldsymbol{X}, \boldsymbol{h}\} = \{\boldsymbol{x}_i, h_i\}_1^N$, initial value of standard deviation σ for Gaussian low-pass filter, stop conditions c_1 and c_2
Output: approximating function $f_k(\boldsymbol{x})$

$\boldsymbol{h}_f = Gauss(\boldsymbol{h}, \sigma)$ `// Gaussian low-pass filter`
$\boldsymbol{\Xi}$ = Compute stationary points of $\{\boldsymbol{X}, \boldsymbol{h}_f\}$ (using algorithm in [10])
$\boldsymbol{\Xi} = \boldsymbol{\Xi} \cup$ (corners of dataset bounding box)
$f_k(\boldsymbol{x})$ = RBF approximation $(\{\boldsymbol{X}, \boldsymbol{h}\}, \boldsymbol{\Xi})$
$\boldsymbol{r}_k = |\boldsymbol{h} - f_k(\boldsymbol{X})|$
$\sigma_k = \sigma$
while $c_1 || c_2$ **do**
    $\boldsymbol{r}_{kf} = Gauss(\boldsymbol{r}_k, \sigma_k)$ `// Gaussian low-pass filter`
    $\boldsymbol{\Xi}_k$ = Compute stationary points of $\{\boldsymbol{X}, \boldsymbol{r}_{kf}\}$ (using algorithm in [10])
    $\boldsymbol{\Xi} = \boldsymbol{\Xi} \cup$ (only local maxima from $\boldsymbol{\Xi}_k$)
    $f_k(\boldsymbol{x})$ = RBF approximation $(\{\boldsymbol{X}, \boldsymbol{h}\}, \boldsymbol{\Xi})$
    $\boldsymbol{r}_k = |\boldsymbol{h} - f_k(\boldsymbol{X})|$
    $\sigma_k = \sigma_k / 2$

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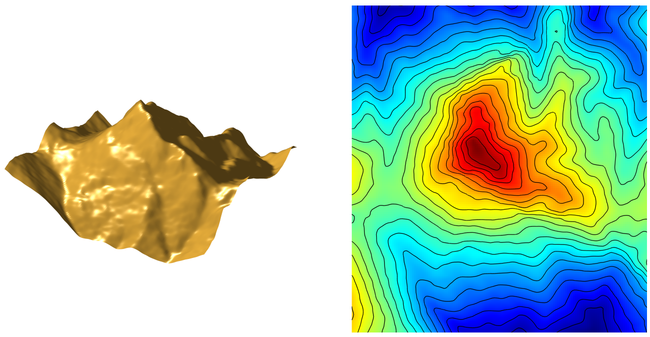
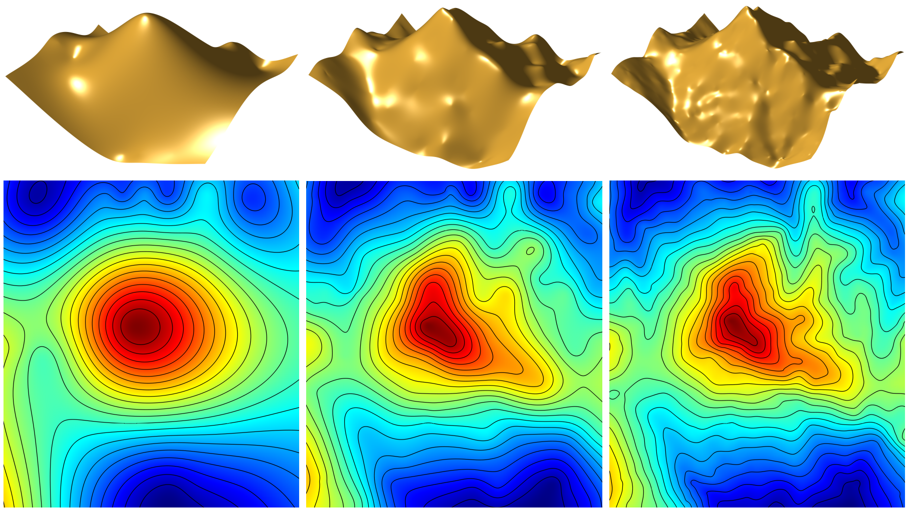
4 Experimental Results

In this section, the experimental results for our proposed approach will be presented. The implementation was performed in Matlab. The thin plate spline (TPS) function $r^2 \log(r^2)$ which is shape parameter free and divergent as radius increases has been used for RBF approximation.

For the purposes of below mentioned experiments, two geographic point clouds were used. The first dataset was obtained from GPS data of the mount Veľký Rozsutec in the Malá Fatra, Slovakia (Fig. 1a) and contains $24,190$ points. The second dataset is GPS data of the part of Pennine Alps, Switzerland (Fig. 2a) and contains $131,044$ points.



(a) original data, $N = 24,190$



(b) 1^{st} level, $M = 19$ (c) 3^{rd} level, $M = 115$ (d) 8^{th} level, $M = 595$

Figure 1. The mount Veľký Rozsutec, Slovakia and its contour map: original data and different levels of proposed incremental RBF approximation when TPS is used.

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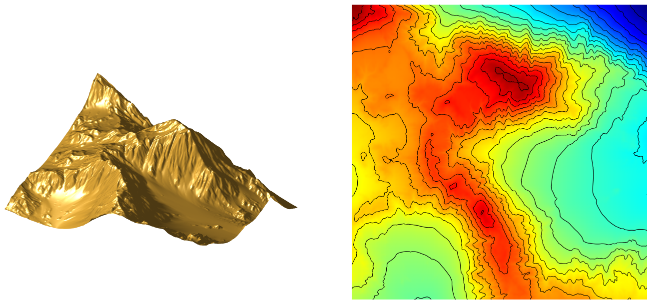
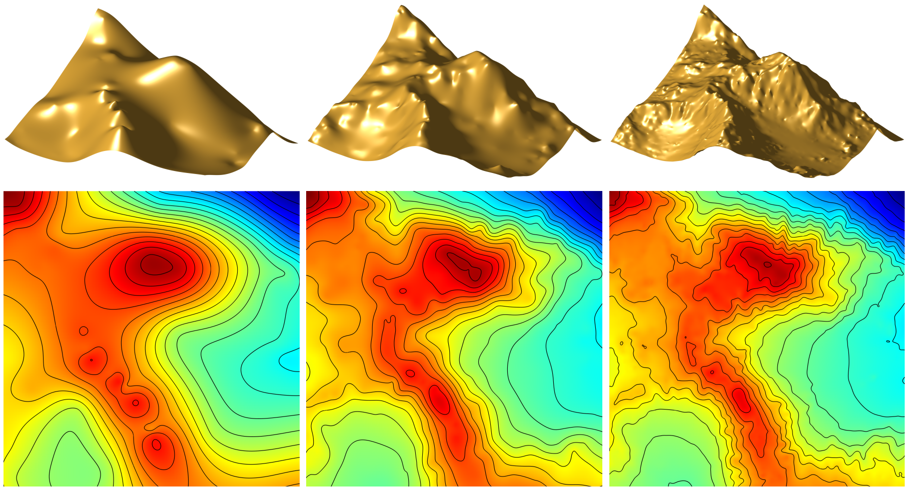
Results for different levels of RBF approximation of the mount Veľký Rozsutec are shown in Fig. 1b - Fig. 1d. We can see that the quality of approximation in terms of error is improving with increasing level of the incremental RBF approximation. For 8^{th} level (see Fig. 1d), the many details of the original terrain are already apparent.

In Fig. 2b - Fig. 2d, the results for different levels of incremental RBF approximation of the part of Pennine Alps are shown. It can be again seen that the quality of approximation is improving with increasing level of the incremental approach. For the first level (see Fig. 2b), it is evident, that the small number of reference points is defined for the ridge in the foreground, and therefore, this ridge is approximated by several peaks in the first level. This problem is eliminated with increasing level of the incremental RBF approximation. For 7^{th} level (see Fig. 2d), the many details of the original terrain are again apparent.



(a) original data, $N = 131,044$



(b) 1^{st} level, $M = 51$ (c) 3^{rd} level, $M = 413$ (d) 7^{th} level, $M = 2602$

Figure 2. The part of Pennine Alps, Switzerland and its contour map: original data and different levels of proposed incremental RBF approximation when TPS is used.

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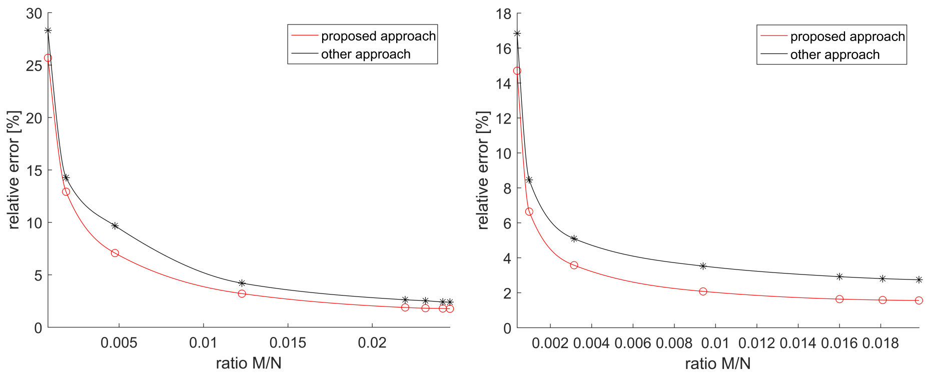

(a) The mount Veľký Rozsutec (b) The part of Pennine Alps

Figure 3. The mean relative error of the proposed incremental RBF approximation in comparison with classical RBF approximation [9] for different compression ratio.

The mean relative error in dependency on compression ratio is presented for both geographic datasets in Fig. 3. Moreover, the comparison of the proposed incremental approach with the classical RBF approximation [9] is performed. From the results, it can be seen that the proposed approach achieves the better quality of results in terms of error.

5 Conclusion

In this paper, a new incremental approach for RBF approximation for geographic data is presented. Selection of the set of reference points for proposed incremental approximation is based on the determination of stationary points of the input point cloud in the first level and the finding local maxima of residues at each hierarchical level. In addition, the Gaussian low-pass filter is used to smooth the trend of the input points, resp. the residues before finding significant points.

The proposed approach achieves the improvement of results in comparison with other existing methods because the features of the given dataset are respected.

In the future work, the proposed approach can be extended to higher dimensions, as the extension should be straightforward. Also, the improving the computational performance without loss of accuracy can be explored.

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