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# 3D Vector Field Approximation and Critical Points Reduction Using Radial Basis Functions

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*Abstract*—Vector field simplification aims to reduce the complexity of the flow by removing features according to their relevance and importance. However, the important features as critical points with a large range of influence should be preserved. We present a new approach for vector field simplification and approximation using Radial basis functions. The experiments proved the ability to approximate complex $3D$ tornado data set. In addition, a significant contribution of the proposed method is also an analytical form of the vector field which can be used in further processing. The abstract goes here.

*Keywords*—Vector field, Radial basis functions, critical point, tornado, simplification, approximation, data compression, visualization.

## I. Introduction

The vector fields are often very large and complex data set. To process and visualize such kind of data, the approximation and simplification techniques are used. The summary of topology based flow visualization techniques is presented in [1]. Most of this techniques tries to simplify the vector topology, either by removing or collapsing critical points, or by smoothing the vector field to remove small unimportant changes in the flow [2], [3], or by simplifying the topological skeleton of the vector field [4], [5].

The paper [6] simplifies the topology of vector field by collapsing critical points. The algorithm processes vector fields defined on a triangulation of the flow domain, i.e. planar vector fields. During the simplification, there are no topological changes in the triangulation. During the simplification are iteratively selected pairs of critical points that can be collapsed. The critical points can be collapsed into one higher order critical point or they can reset each other, i.e. both critical points have opposite Poincare index. Another approach is presented in [7]. There is defined some maximal distance of two critical points to collapse them. All critical points inside the selected radius are collapsed into one higher order critical point. The paper [8] focuses on visualization of complex $3D$ vector fields. The authors prove that vector field inside some area can be described with the $2D$ vector field on the surface around this area. Using this knowledge, they create symbols and visualization based on different behavior of the vector field inside some area with many critical points. The paper [9] presents an approach for simplification of vector field topology, while preserving important features of the vector field, i.e. critical points and separatrices. Each topology feature gets computed a weight that means the importance. According this weights is then simplified the vector field topology. The vertex deletion from the Delaunay triangulation is used in [10] to simplify the vector field. Based on some metrics is determined if a vertex can be removed from the Delaunay triangulation, so that the vector field will not change significantly. The authors prove that this can be determined using only local neighbors and there is no need to compute this change for the whole vector field. The paper [11] filters out the less important and sometimes even sporadic critical points. The filtering is based on the vector field vorticity and is best suited for regional climate modeling and simulation.

Many approaches for vector field approximation use the Radial basis function (RBF) method [12], [13], [14]. The paper [15] presents an approach for large scattered $3D$ vector field approximation. It uses the space subdivision to process and speed-up the approximation. The comparision of vector field approximation with local radial basis functions and global radial basis functions is presented in [16]. The paper [17] uses the RBF interpolation in numerical simulation of divergence-free vector fields. Another approaches use the second order derive to describe features of the vector field [18], [19].

## II. Proposed Approach

The $3D$ vector field data sets come usually from numerical simulations and are very large. Such vector fields can be approximated for the visualization purposes or to minimize the data set size. In our proposed approach, we use modified algorithm described in [20] to approximate the $3D$ vector fields. In this paper, we will especially focus on approximation of the EF5 tornado data set (from [21])[1].

The $3D$ data set is divided into $2D$ horizontal slices, as the main swirl plane is horizontal. Each $2D$ vector field slice contains high number of critical points that we want to reduce. We will use the algorithm described in [20] to determine the important critical points. The important critical points should be presented in the final approximated vector field, while the unimportant critical points should be removed from the vector field. The important critical points will be the centers for radial basis functions (RBF) [22]. The radial basis functions should

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[1] Data set of EF5 tornado courtesy of Leigh Orf from Cooperative Institute for Meteorological Satellite Studies, University of Wisconsin, Madison, WI, USA.

be also placed at the extremes of $v_x$, $v_y$, resp. $v_z$. However the number of extremes is too high and thus this extremes need to be reduced. For this purpose, we use the Gaussian low-pass filter to determine only the important extremes and to discard the small local extremes. The last additional centers of RBF are located at the data set bounding box vertices. The vector field is then approximated using RBF as

$$\boldsymbol{v}(\boldsymbol{x}) = \sum_{j=1}^{M} \boldsymbol{\lambda}_j \varphi(\|\boldsymbol{x} - \boldsymbol{\xi}_j\|) \tag{1}$$

where $\boldsymbol{\lambda}_j$ are weights of the RBFs, $M$ is the number of the radial basis functions, $\varphi$ is the radial basis function and $\boldsymbol{\xi}_j$ are centers of radial basis functions. It is similar as in the potential field case [23]. For a given vector field data set $\{\boldsymbol{x}_i, \boldsymbol{v_i}\}_1^N$, where $N \gg M$, the following overdetermined linear system of equations is obtained:

$$\boldsymbol{v}_i = \boldsymbol{v}(\boldsymbol{x}_i) = \sum_{j=1}^{M} \boldsymbol{\lambda}_j \varphi(\|\boldsymbol{x}_i - \boldsymbol{\xi}_j\|) \quad \text{for } \forall i \in \{1, \ldots, N\}. \tag{2}$$

The result of the RBF approximation is an analytical description of the $3D$ vector field. This is the advantage over other existing methods that use the triangulation, resp. tetrahedronization. The analytical description can be used for further processing of the vector field.

## III. Experimental Results

We tested the proposed approach using the EF5 tornado data set from [21]. For the testing purposes, we selected the central part of the data set, where the tornado is located. The size of the vector field data set for approximation is $8 \cdot 10^6$ points (see Fig. 4a).

First step of the proposed approach is the reduction of critical points in horizontal slices of the vector field. The input data set contains 28 902 critical points (see Fig. 1a) and after reduction using algorithm [20], we end up with only 490 critical points (see Fig. 1b).

The centers of the RBFs are located at critical points and at the extremes of $v_x$, $v_y$, resp. $v_z$. After smoothing using Gaussian low-pass filter, we located 666 extremes. The total number of centers for radial basis functions is thus 1164 points (see Fig. 2).

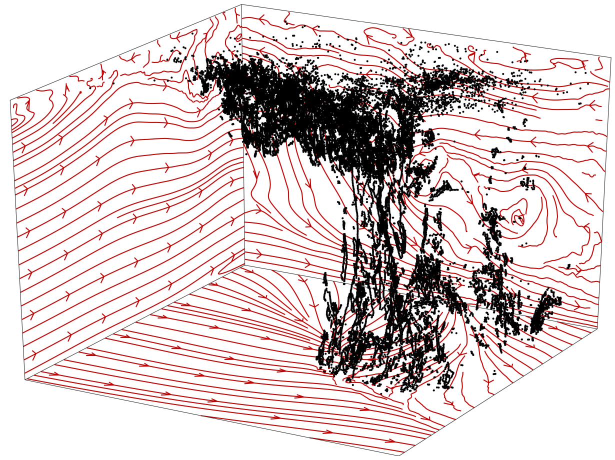
The data set is very large for RBF approximation, thus we need to use the local RBF [24], [25], [26] to reduce needed memory (see [20] for selection of RBF). The RBF used for vector field approximation is

$$\varphi(r) = (1 - r)_+^4 (4r + 1) \tag{3}$$

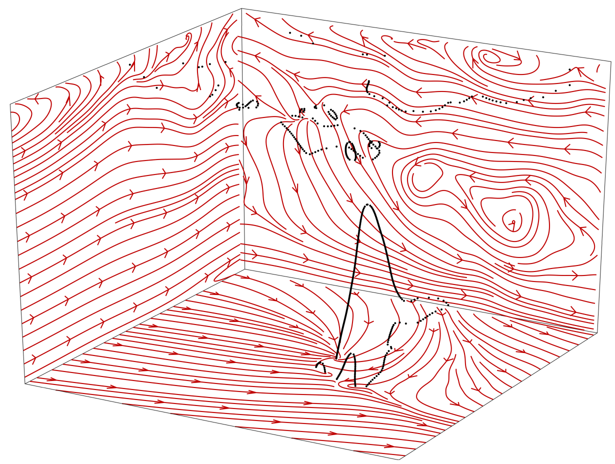
as it gives the best approximation results based on experimental results and is $C^2$ continuous, which is appropriate when computing derivatives of the vector field.

After the RBF approximation of the input vector field data set, we visualized the results. The $2D$ vector field horizontal slices are visualized in Fig. 3. It can be seen, that the global character of the vector field remains the same. There are only local differences as the approximated vector field is smoother. There can be seen some small differences in the global direction of the flow, however the compression ratio is about $7 \cdot 10^3 : 1$ which is very high.

Fig. 4 presents the visualization of $3D$ approximated vector



(a) All critical points.



(b) Reduced critical points.

Fig. 1. Visualization of $2D$ critical points located at horizontal slices of vector field data set (28 902 points (a) and 490 points (b)).

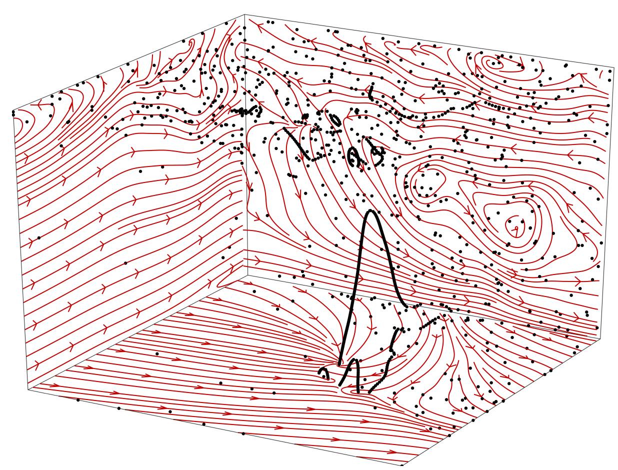


Fig. 2. Visualization of centers of radial basis functions (1164 points).

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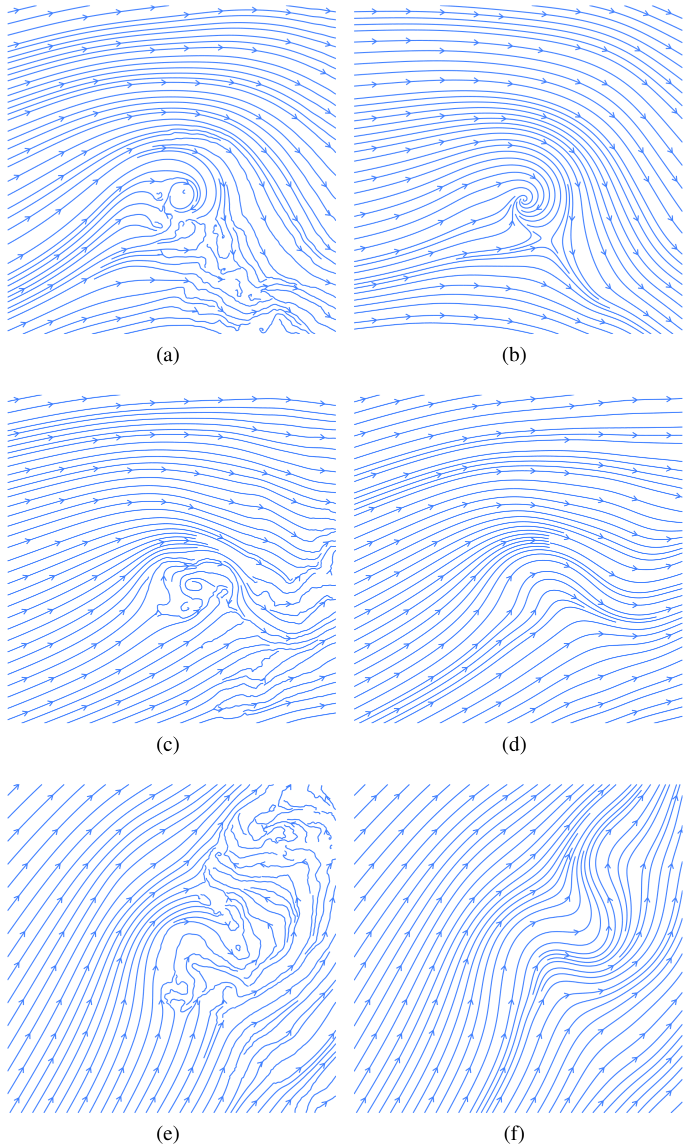
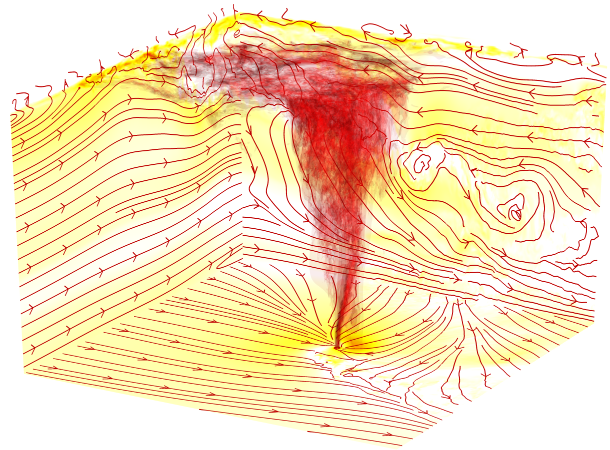
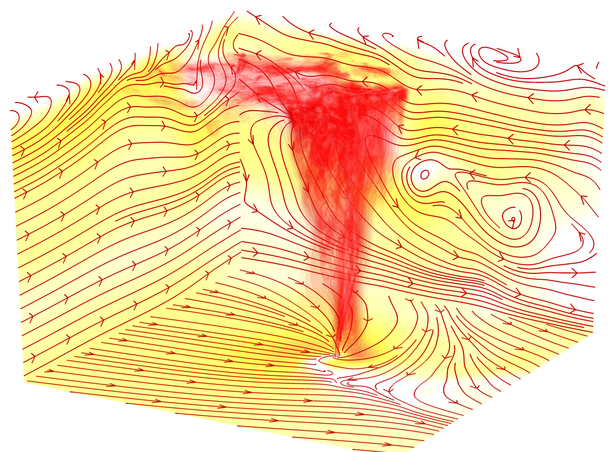


Fig. 3. Visualization of $2D$ vector field horizontal slices. Left column represents the original vector field and the right column represents the RBF approximated vector field.



(a) Original vector field.



(b) Approximated vector field.

Fig. 4. Visualization of the $3D$ tornado vector field data set. Red central part represents the shape of tornado vortex and the yellow color on faces represents the speed of vector field.

field. It can be seen that the main vortex of the tornado has similar shape to the original one. The original one contains just many tiny details. However this is again due to very high compression ratio ($7 \cdot 10^3 : 1$).

The approximation error can be measured using different formulas. The first way is to compute the average difference of the approximated vector field and the original vector field. The average difference is computed using

$$Err = \frac{\sum_{i=1}^{N} \|\boldsymbol{v}_i - \bar{\boldsymbol{v}}_i\|}{N}, \tag{4}$$

where $\boldsymbol{v}_i$ is the approximated vector, $\bar{\boldsymbol{v}}_i$ is the original vector and $N$ is the number of the original samples. The approximation error is visualized in Fig. 5a.

Next, we can measure the average vector length error, i.e. the average speed error. This error is computed using

$$Err = |\|\boldsymbol{v}_i\| - \|\bar{\boldsymbol{v}}_i\||\,. \tag{5}$$

The computed speed error is visualized in Fig. 5b. The speed error of vector field approximation is $3.8\ ms^{-1}$ and the average speed of the vector field is $18.7\ ms^{-1}$.

## IV. Conclusion

We presented a new approach for simplification and approximation of complex and large $3D$ vector fields using RBF. The proposed approach preserves during the simplification and approximation the important critical points and thus it preserves the global character of the $3D$ vector field as well. As the result, we end up with an analytical description of the approximated vector field, which can be used for further processing of the simplified vector field.

In the future, the proposed approach will be modified to reduce the critical points already in $3D$ instead of in $2D$ vector field slices.

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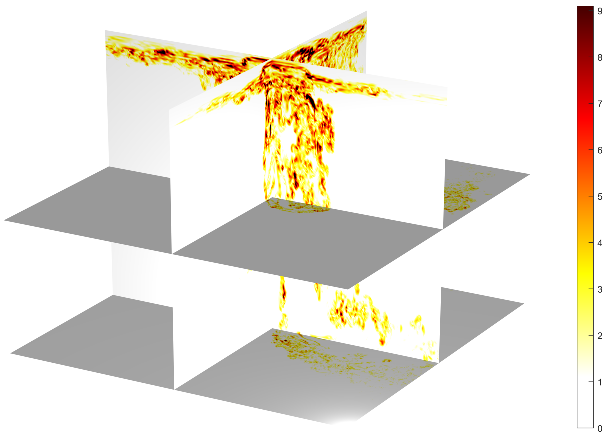
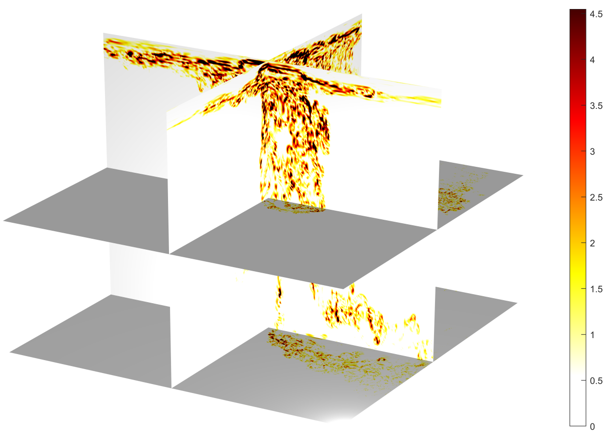



(a) Approximation error.




(b) Speed error.

Fig. 5. Visualization of vector field approximation error (a) and visualization of speed error of approximated vector field (b).

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