Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems, ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems *

Vaclav Skala[1] [0000−0001−8886−4281]

University of West Bohemia, Faculty of Applied Sciences
Dept. of Computer Science and Engineering
Pilsen, CZ 301 00, Czech Republic
skala@kiv.zcu.cz www.VaclavSkala.eu

Abstract. Polygonal meshes are used in CAD/CAM systems and in solutions of many of engineering problems. Many of those rely on polygonal representation using facets, edges and vertices. Today, due to numerical robustness as only three points can lie on a plane, limited numerical precision of the floating point representation, etc. the triangular facets are used nearly exclusively. This is a significant factor witch is not fully considered in triangular mesh representations and their processing.
This contribution presents a new approach to the 3D geometric model representation based on vertices and edges only, i.e. by the wire-frame data model, where no facet representation is needed, if the surface is formed by a triangular mesh.
The wire-frame representation use leads to significant reduction of data as there is no need to represent facets explicitly. It can be used for significant data compression, etc. Examples demonstrating the worst cases solutions are presented with a 3D print of those.

Keywords: Polygonal model · edge model · wire-frame model · polygonal meshes · triangular mesh · surface model consistency · 3D geometric modeling · surface representation · data compression · mesh surface reconstruction · CAD/CAM systems · manufacturing · 3D print · geometry hash function.

1 Introduction

Geometric modeling of 3D surfaces is based on polygonal meshes, parametric surfaces or implicitly defined objects in the vast majority of cases. In the case of polygonal meshes a surface is represented by a polygonal mesh Botsch[6], Wenger[28], Apostol[2], Brentzen[7], Levy[21] which is usually represented by data structures as half-edge, winged-edge, etc.

---

* Research supported by the University of West Bohemia - Institutional research support No.1311.

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems, ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

A significant effort is devoted to:

- polygon mesh repair Attene[4],
- mesh reconstruction from a set of triangles Glassner[12], Skala[25],
- memory optimization and mesh registration Le Clerc[19],
- hole filling in manifold meshes Gou[14],
- Boolean operations on polyhedral meshes Landier[18], Fu[11], Attene[3] and intersection problems Held[15],
- surface reconstruction from sketched drawings Company[9], Yu[32], or projections Lee[20], Alexei[1], Shin[24], Yan[31] and Fang[10].

A wire-frame representation is not considered as it does not represent a surface unambiguously in the case of polygonal faces Varley[26], Botsch[6].

However, several attempts for reconstructions of 3D solid objects were made by Mirandi[23], Markowsky1980[22] and Zabran[33], Inouse[17] and Bandla[5]. Necessity of facets in the surface representations is presented in nearly all books on computer graphics and geometric modeling.

However nowadays, the polygonal meshes are nearly exclusively represented by triangular meshes, as the only three points can lie on a plane due to limited numerical precision due to use of the IEEE-754[29] floating point representation. This significant property has important consequences in many aspects, e.g. can lead to decrease of memory requirements, simpler data structures for data transmission, higher data compression ratio and to higher robustness.

In the following, a new robust method for triangular 3D surface mesh reconstruction from the wire-frame data model, i.e. object is only represented by vertices and edges, as a consequence of the triangular representation. This breaks the "axiom" on the ambiguity of 3D wire-frame models representation in the case of triangular meshes used for a surface representation of geometrical objects.

The proposed approach can also lead to new algorithms for geometric model consistency check, data compression for data transmission etc. Also, it can be extended for general polygonal meshes and for manifold objects using "virtual edges".

2 Data Structures

Mesh based data structures have several variants used in applications. They have to be evaluated especially from requirements and their properties point of view, e.g. algorithmic, topological, etc., see WiKi[30] The mesh data structures can be classified to:

- Face-based structures, which is a set of "unconnected" triangles. It is a simple data structure, where no information on neighbor triangles is directly available. Typical example is the STL format used in CAD/CAM systems for 3D printing. Non-trivial algorithms for a mesh reconstruction are to be used Glassner[13], Hradek[16] and Skala[25]
- Edge-based structures, where an edge is the edge shared by two polygons. The typical example is the winged-edge structure [Baumg72]

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems, ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

– Half-edge structure, which is based on splitting an edge to two oriented half-edges. More efficient modification was introduced in Campagna[8]

All data structures have several variants reflecting specific requirements from targeted application, the algorithmic point of view, memory requirements, consistency and robustness aspects etc. However, in some cases memory requirements are critical, e.g. in data compression and transmission, surface reconstruction from acquired data etc.

The CAD/CAM systems use different surface representations of geometrical objects and non-trivial operations with them, q.e. union, difference, etc. There are still problems with robustness and correctness of those operations, see Wassermann[27] as those issues are critical for the CAD/CAM systems.

The wire-frame data model, which contains only information on vertices coordinates and edges, is not defining the surface unambiguously, if the polygonal model is used, i.e. faces are not triangulated. The Fig.1 presents the wire-frame model and three different interpretations of the wire-frame model, if polygonal mesh is used (triangulation of the faces is not considered).

It can be seen, that the holes can be front-back, left-right or top-bottom oriented in the case that the faces are un-triangulated polygons. This a typical example of an ambiguity of the wire frame model, in the case of the polygonal representation.

However, in the case of the objects with a triangular mesh, the triangular property of the given surface is so significant, that should have substantial influence to the algorithms and data structures, which represent a surface of the given object.

There is a question, whether a surface based on a triangular mesh can be represented unambiguously by vertices and edges only, i.e. by a wire-frame data model.

3 Wire-frame data model

Data structures and algorithms used in polygonal meshing are widely described with many specific modifications. The wire-frame data structure, which contains only vertices coordinates and edges, is considered as ambiguous. In the case of polygonal meshes the ambiguity is clearly visible on the “4D-Cube” data model, see Fig.1.

In the case of simple polygonal meshes which:

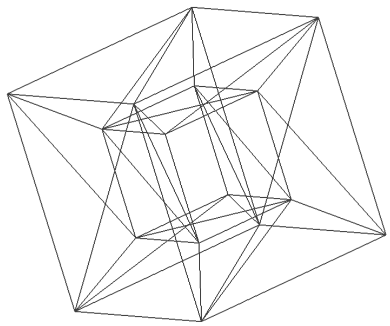
– do not have a hole, i.e. Genus zero
– are not self-intersecting, not touching on a vertex, edge and face
– surface is ”water proof”

the simple algorithm based on “pairing” is quite simple and applicable for the most cases. It is based on finding cycles in a graph of the given edges of the length 3. However, such algorithm fails on very simple and similar cases, see Fig.2, Fig.3 and Fig.4.

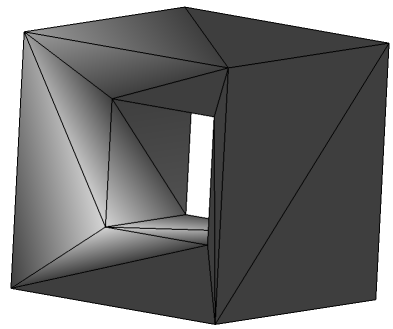
The situation gets more complicated in the case of objects with holes, which is a quite common case of models originated from CAD/CAM systems.

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

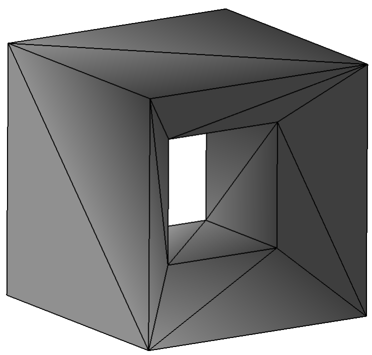
Fig. 1: Different possible interpretations of the 4D-Cube wire-frame and shaded models



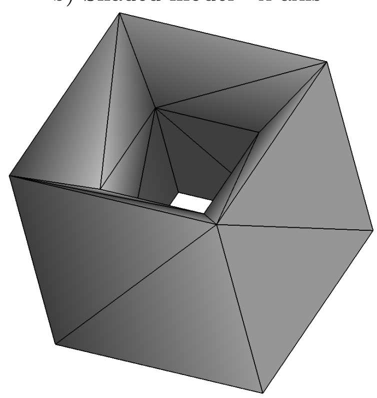
a) Wire-Frame model



b) Shaded model - x axis

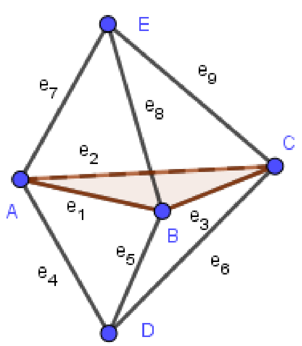


c) Shaded model - y axis

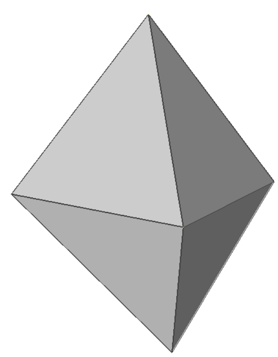


d) Shaded model - z axis

Fig. 2: Join of two tetrahedra



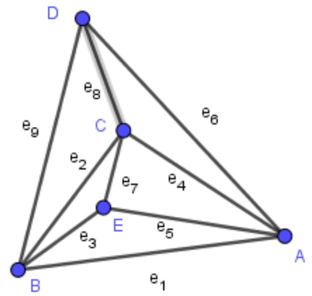
a) Wire-Frame of two tetrahedra



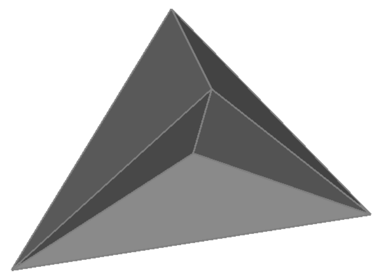
b) Shaded two tetrahedra

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems, ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

Fig. 3: Difference of two tetrahedra



a) Wire-Frame of two tetrahedra



b) Shaded two tetrahedra

4 Wire-frame for triangle meshes

The surfaces of geometric models are represented by triangular meshes nearly exclusively. It leads to better numerical precision of computation, etc.

As the knowledge, that the mesh is a triangular mesh, is so significant, that it should lead to significant data structure simplification or to a new data structure itself.

In the case of the triangular mesh it is expected that:

- a 3D surface is closed (but not necessarily)
- a surface of 3D object represented by a triangular mesh
- 3D surface is "water proof"
- each triangle is a triangle of the final surface
- any edge is shared by two triangles only (if the 3D surface is closed)
- 3D surface is non-manifold

When the object at Fig.1 is represented by a triangular mesh, the correct surface can be generated. However, in some even simple cases, the algorithm of surface generation can be more complicated.

4.1 Non-trivial cases without holes

The surface generation of 4D cube model at Fig.1 proofs that restriction to triangular mesh representation is beneficial. However, such knowledge does not guarantee correctness of the surface generated.

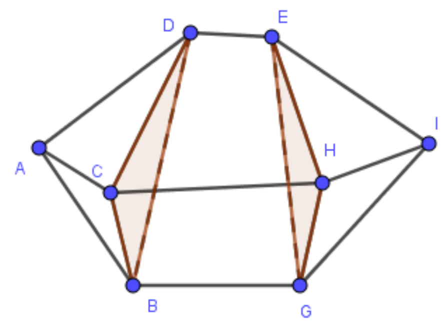
Let us consider Computer Solid Geometry (CSG) representation and tetrahedrons T_{ABCD}, T_{ABCE} and T_{ABCF}. Then, the object at Fig.2 is given as $T_{ABCD} \cup T_{ABCE}$, i.e. union operation, while the object at Fig.3 is given as $T_{ABCD} - T_{ABCE}$, i.e. difference operation. The "pairing algorithm" generates a triangle T_{ABC}, which is an invalid as it is a residue from the CSG operation.

In the both cases, the simple algorithm fails, as it would generate a non-existent common face of those two tetrahedrons. In the first case, such incorrect

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

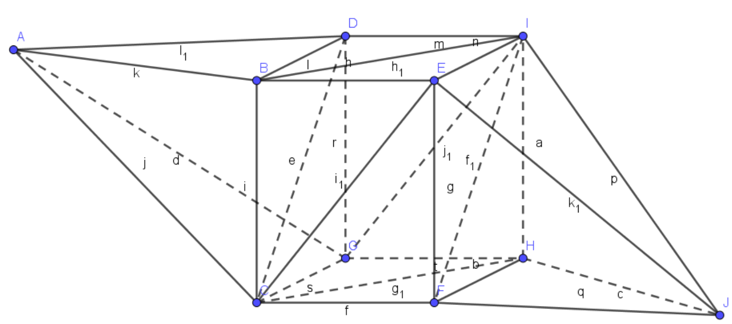
face would be inside, in the second case, the invalid case would “close” the hole of the object at Fig.3. On the other hand, some objects might have a hole similarly as in the “4D-Cube” case at Fig.1. In this case, such simple algorithm would delivered correct result, if the wire frame model represents the triangular mesh. Actually it is due to a non-triangular cross section of the object. The simple algorithm would also fail in the case of a prism with a triangular profile, see Fig.4. In this case also non-existent triangles in side of the object would be generated. It can be seen, that these simple examples demonstrate, that such simple algorithm would not work properly.

Fig. 4: Prism



However, in the case Fig.5, the simple algorithm generates a correct triangular mesh. It can be seen that the triangular non-existent triangles or triangular holes causing problems in the correct triangular mesh generation.

Fig. 5: Prism-rectangular-profile

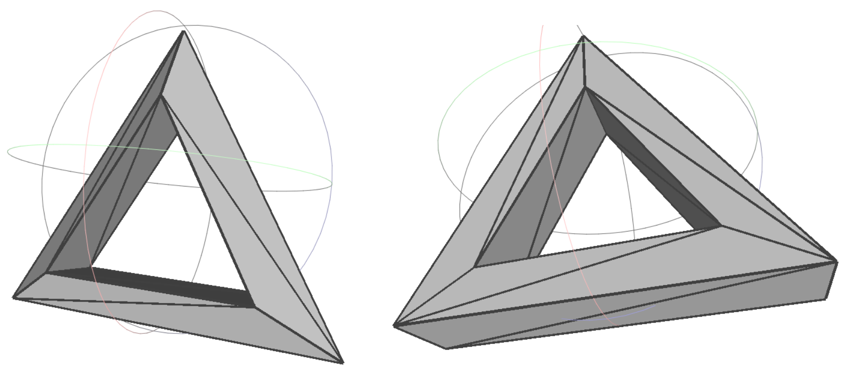


Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

4.2 Non-trivial cases with holes

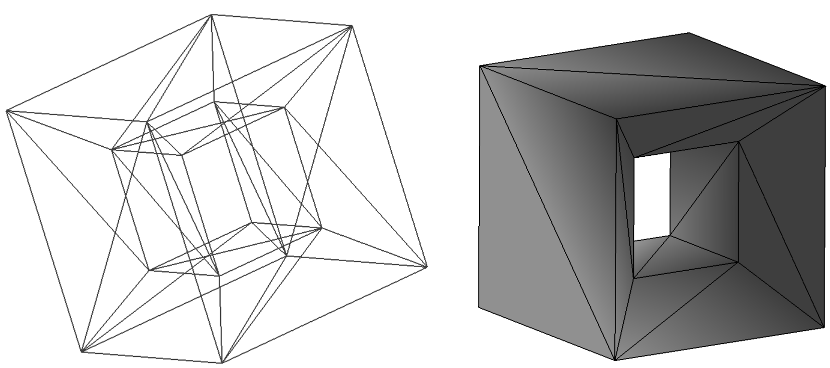
The "triangular-ring" model at Fig.6 represents the case, when "virtual triangles" at the corners can be generated from the wire-frame model, but they are not part of the surface of those objects, which are actually non-manifold unions of three prisms with triangular, Fig.6.a, and non-triangular, Fig.6.b, cross sections of the objects.

Fig. 6: Triangular ring



a) Ring with a triangular profile b) Ring with a squared profile

Fig. 7: 4D Cube testing model



a) 4D cube wire-frame b) 4D cube shaded

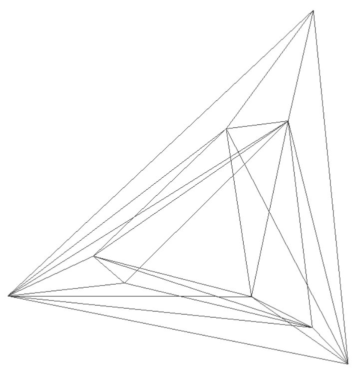
There are two main problems in unambiguity of the surface reconstruction given by the wire-frame model, if general polygonal faces:

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

- Ambiguous surface as in the case of the “4D-Cube” model, see Fig.7
- Non-existent (fictive) faces generation as in the case of the “Triangular-ring” model, where 3 non-existent (inner) triangles would be generated on the intersection of two triangular prisms, see Fig.6.

In the case of a triangular ring with the triangular profile, see Fig.8, is probably the most simple object causing the most problems in correct surface generation as 4 false triangles would be generated by a naive simple algorithm.

Fig. 8: 3D triangular ring testing model



a) 3D ring wire-frame

b) 3D ring shaded

If a surface is represented as a triangular mesh, it gives us significant additional information, which should be used to decrease data, i.e. memory, requirements and also can lead to new algorithms for consistency checks, data compression, transmission, etc.

The unambiguous surface generation of the “triangular-ring” wire-frame model is possible, if triangular mesh is used, see Fig.8-b.

In the following the principle of the proposed algorithm will be described.

5 Proposed algorithm

Let us consider a simple object which is result of a union operation on two tetrahedra Fig.9.

The wire-frame data structures representing this polyhedron is given by the table of edges TAB.1 and by the table of vertices coordinates x, y, z TAB.2.

For the surface reconstruction from the wire-frame data model, actually only a table of edges is required, as vertex coordinates are not used in manifold single triangular surface model cases.

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems, ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

Fig. 9: Example - two joint tetrahedra

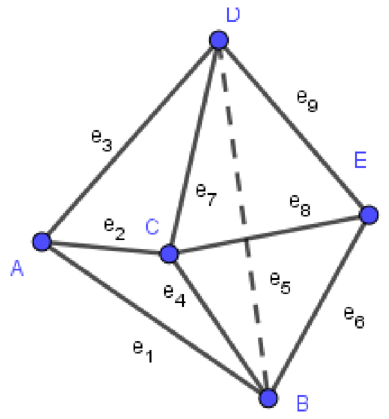


Table 1: Wire-frame data structure - edges

ID_E	1	2	3	4	5	6	7	8	9
V_1	1	1	1	2	2	2	3	3	4
V_2	2	3	4	3	4	5	4	5	5

Table 2: Wire-frame data structure - vertices

	A	B	C	D	E
ID_V	1	2	3	4	5
x	0	3	1	2	4
y	0	-2	1	2	-2
z	0	2	1	-1	3

It should be noted that ID_E, resp. ID_V are identifiers of edges, resp. vertices and are not actually part of data structures necessarily.

The proposed algorithm is actually very simple in principle for the closed surfaces.

1. Find all triangles sharing common edge twice only, generate those triangles and sign edges of those triangles as used
   and update data structures reflecting those changes
2. Find all triangles having the only one common edge shared three times, generate those triangles and sign edges of those triangles as used
   and update data structures reflecting those changes
3. Find all triangles having the only two common edges shared three times, generate those triangles and sign edges of those triangles as used
   and update data structures reflecting those changes
4. Find all triangles having all three common edges shared three times, generate those triangles and sign edges of those triangles as used

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

and remove such a triangles from the set of given reminding triangles as those triangles are not part of the surface.

In the case, that the that the surface is not closed, i.e. it has the border, additional check is to be used to verify that the borders forms closed boundary curve.

The result of the algorithm is a non-ordered set of triangles, i.e. there is no information on the triangle neighbours. However, the basic algorithm can be modified and the triangular mesh can be generated, i.e. information of neighbours is generated as well.

It should be noted, that the reconstruction of a triangular mesh from a set of triangles is a nontrivial task. Triangular mesh reconstruction from a set of triangles is described in Glassner[13], Hradek[16] and Skala[25].

6 Experimental results

The proposed algorithm was tested for several data sets, some of those were specifically constructed to "attack" the proposed algorithm, e.g.

- A "ring" shape polyhedron which has a triangular profile, see Fig.7
- A "4D-Cube" model with triangular faces, see Fig.6 , Fig.8
- Polyhedron with 6 faces formed as a union of two tetrahedrons with a "common" face, see Fig.2, Fig.3, Fig.9, etc.

The resulting triangular meshes were stored in the PLY format and 3D printed. The Fig.10 presents 3D print of the reconstructed "4D-Cube" and "triangular ring" data.

The experiments proved functionality of the proposed algorithm and correctness of its implementation in Pascal using Lazarus IDE.

The proposed algorithm can be easily modified for the case when some triangles on the surface of a model are missing. In this case the input data must contain information on those triangles in explicit form and they will be handled as non-existent virtual triangles.

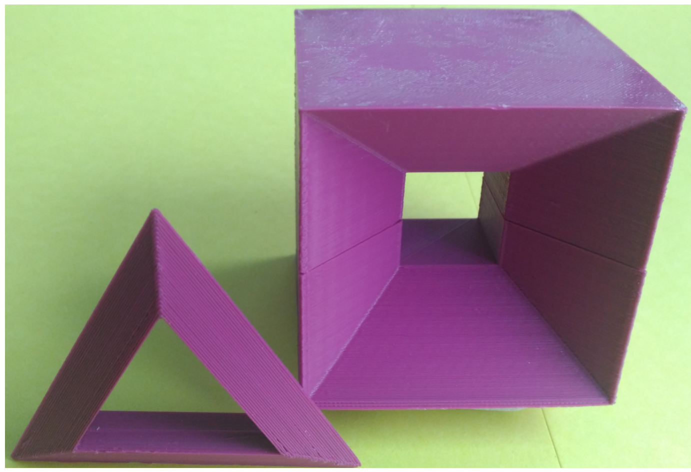
7 Conclusion

Triangular meshes are used for surface models nearly exclusively due to many reasons. However, the triangular mesh properties have not been fully explored yet as far as the unambiguity mesh representation, storing, compression and transmission, etc. This paper presents a new algorithm, which enables to reconstruct faces of a triangular mesh surface from wire-frame data structure, which was considered as ambiguous representation in computer graphics, CAD/CAM systems.

The proposed algorithm breaks the "axiom" on the ambiguity of wire-frame 3D models representation in the case of triangular meshes. The proposed approach can also lead to new algorithms for mesh consistency checks, data compression applicable especially in CAD/CAM systems and transmission, etc. The

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

Fig. 10: 3D print of "4D-Cube" and "triangular ring" reconstruction from the wire-frame model



proposed algorithm can reconstruct a surface of non-manifold objects is they share a common face, similarly as in the "triangular-ring" model.

Acknowledgment

The author would like to thank to recent students at the VSB-Technical University and Ostrava University, University of West Bohemia as the method was designed during the Fall term in 2013 and verified by experimental implementations in Pascal.

Thanks also belong to colleagues at the University of West Bohemia and Shandong University (China) for their valuable comments, hints provided and for printing 3D model and to anonymous reviewers for their critical views and hints leading to significant paper improvement.

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems, ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

References

1. Alexei, Z., Arkady, Z.: Three-dimensional reconstruction from projections based on incidence matrices of patterns. AASRI Procedia **9**, 72–77 (2014). https://doi.org/https://doi.org/10.1016/j.aasri.2014.09.013, https://www.sciencedirect.com/science/article/pii/S2212671614001139, 2014 AASRI Conference on Circuit and Signal Processing (CSP 2014)
2. Apostol, K.: Polygonal Modeling. SaluPress (2012). https://doi.org/10.5555/2463089
3. Attene, M.: Indirect predicates for geometric constructions. CAD Computer Aided Design **126** (2020). https://doi.org/10.1016/j.cad.2020.102856
4. Attene, M., Campen, M., Kobbelt, L.: Polygon mesh repairing: An application perspective. ACM Comput. Surv. **45**(2) (Mar 2013). https://doi.org/10.1145/2431211.2431214
5. Bandla, S., Gurumoorthy, B.: Constructing a wire-frame from views on arbitrary view planes for objects with conic sections inclined to all view planes. CAD Computer Aided Design **43**(12), 1639–1653 (2011). https://doi.org/10.1016/j.cad.2011.08.010
6. Botsch, M., Kobbelt, L., Pauly, M., Alliez, P., Levy, B.: Polygon Mesh Processing. CRC Press (2010)
7. Brentzen, J.A., Gravesen, J., Anton, F., Aans, H.: Guide to Computational Geometry Processing: Foundations, Algorithms, and Methods. Springer (2012). https://doi.org/10.5555/2349037
8. Campagna, S., Kobbelt, L., Seidel, H.P.: Directed edges—a scalable representation for triangle meshes. J. Graph. Tools **3**(4), 1–11 (Dec 1998). https://doi.org/10.1080/10867651.1998.10487494
9. Company, P., Varley, P., Plumed, R., Camba, J.: Detection of vertices in sketched drawings of polyhedral shapes. LNCS **11401 LNCS**, 376–383 (2019). https://doi.org/10.1007/978-3-030-13469-3_44
10. Fang, F., Lee, Y.: 3d reconstruction of polyhedral objects from single perspective projections using cubic corner. 3D Research **3**(2), 1–8 (2012). https://doi.org/10.1007/3DRes.02(2012)1
11. Fu, Z.G., Zou, B.J., Chen, Y.M., Wu, L., Shen, Y.: Reconstruction of intersecting curved solids from 2d orthographic views. CAD Computer Aided Design **42**(9), 841–846 (2010). https://doi.org/10.1016/j.cad.2010.03.005
12. Glassner, A.: Building Vertex Normals from an Unstructured Polygon List, p. 60–73. Academic Press Professional, Inc., USA (1994)
13. Glassner, A.: Building vertex normals from an unstructured polygon list. Graphics Gems IV pp. 60–73 (1994)
14. Gou, G., Sui, H., Li, D., Peng, Z., Guo, B., Yang, W., Huang, D.: Limofilling: Local information guide hole-filling and sharp feature recovery for manifold meshes. Remote Sensing **14**(2) (2022). https://doi.org/10.3390/rs14020289, https://www.mdpi.com/2072-4292/14/2/289
15. Held, M.: Erit—a collection of efficient and reliable intersection tests. Journal of Graphics Tools **2**(4), 25–44 (1997). https://doi.org/10.1080/10867651.1997.10487482
16. Hrádek, J., Kuchař, M., Skala, V.: Hash functions and triangular mesh reconstruction. Computers and Geosciences. **29**(6), 741–751 (2003). https://doi.org/10.1016/S0098-3004(03)00037-2

Skala,V. A novel, fast and robust triangular mesh reconstruction from a wire-frame 3D model with holes for CAD/CAM systems,
ICCSA 2022, LNCS 13375, pp. 399-411, 2022, ISBN 978-3-031-10522-7, Springer, 2022
DOI: 10.1007/978-3-031-10522-7_28

17. Inoue, K., Shimada, K., Chilaka, K.: Solid model reconstruction of wire-frame cad models based on topological embeddings of planar graphs. Journal of Mechanical Design, Transactions of the ASME **125**(3), 434–442 (2003). https://doi.org/10.1115/1.1586309
18. Landier, S.: Boolean operations on arbitrary polygonal and polyhedral meshes. CAD Computer Aided Design **85**, 138–153 (2017). https://doi.org/10.1016/j.cad.2016.07.013
19. Le Clerc, F., Sun, H.: Memory-friendly deep mesh registration. CSRN-Computer Science Research Notes **3001**(1), 1–10 (2020). https://doi.org/10.24132/CSRN.2020.3001.1
20. Lee, Y., Fang, F.: A new hybrid method for 3d object recovery from 2d drawings and its validation against the cubic corner method and the optimisation-based method. CAD Computer Aided Design **44**(11), 1090–1102 (2012). https://doi.org/10.1016/j.cad.2012.06.001
21. Levy, B., Zhang, H.R.: Elements of geometry processing. In: SIGGRAPH Asia 2011 Courses. pp. 1–48. SA '11, ACM, New York, NY, USA (2011). https://doi.org/10.1145/2077434.2077439
22. Markowsky, G., Wesley, M.A.: Fleshing out wire frames. IBM Journal of Research and Development **24**(5), 582–597 (1980)
23. Minardi, P.: Reconstruction of 3-dimensional solid objects represented by wire-frame descriptions. In: Earnshaw, R.A., Wyvill, B. (eds.) New Advances in Computer Graphics. pp. 489–503. Springer Japan, Tokyo (1989)
24. Shin, B.S., Shin, Y.: Fast 3d solid model reconstruction from orthographic views. CAD Computer Aided Design **30**(1), 63–76 (1998). https://doi.org/10.1016/S0010-4485(97)00054-7
25. Skala, V., Kuchař, M.: The hash function and the principle of duality. Proc. Computer Graphics Int. CGI2001 pp. 167–174 (2001)
26. Varley, P.A., Company, P.P.: A new algorithm for finding faces in wireframes. Computer-Aided Design **42**(4), 279–309 (2010). https://doi.org/10.1016/j.cad.2009.11.008
27. Wassermann, B., Kollmannsberger, S., Yin, S., Kudela, L., Rank, E.: Integrating cad and numerical analysis: 'dirty geometry' handling using the finite cell method. Computer Methods in Applied Mechanics and Engineering **351**, 808–835 (2019). https://doi.org/10.1016/j.cma.2019.04.017
28. Wenger, R.: Isosurfaces: Geometry, Topology, and Algorithms. A.K.Peters/CRC Press, New York (2013). https://doi.org/10.1201/b15025
29. Wikipedia: IEEE 754 - Wikipedia, the free encyclopedia (2021), https://en.wikipedia.org/wiki/IEEE_754, [Online; accessed 11-July-2021]
30. Wikipedia contributors: Polygon mesh - Wikipedia, the free encyclopedia (2021), https://en.wikipedia.org/wiki/Polygon_mesh, [Online; accessed 19-October-2021]
31. Yan, Q.W., Philip Chen, C., Tang, Z.: Efficient algorithm for the reconstruction of 3d objects from orthographic projections. Computer-Aided Design **26**(9), 699–717 (1994). https://doi.org/10.1016/0010-4485(94)90020-5
32. Yu, H., He, Y., Zhang, W.: A new approach to analyzing interactions of two objects in space based on a specially- tailored local coordinate system. IEEE Access **9**, 60258–60264 (2021). https://doi.org/10.1109/ACCESS.2021.3074509
33. Zabran, M.: An algorithm recreating 3d triangle mesh faces from its edges. CESCG 2018 pp. 1–8 (2018), https://cescg.org/wp-content/uploads/2018/04/Zabran-An-Algorithm-Recreating-3D-Triangle-Mesh-Faces-from-Its-Edges.pdf