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Hermite Parametric Bicubic Patch Defined by the Tensor Product *

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Abstract. Bicubic parametric plates are essential for many geometric applications, especially for CAD/CAM systems used in the automotive industry, mechanical and civil engineering applications. Usually the Hermite, Bézier, Coons or NURBS plates are used. There is always a problem to explain how the Hermit bicubic plate is constructed. This contribution describes a novel formal approach to Hermite bi-cubic plate construction using the tensor product.

Keywords: Hermite curve · Hermite bicubic patch · interpolation · tensor product · parametric patches · Kronecker product.

1 Introduction

This contribution describes a novel approach for deriving Hermite bicubic parametric patch. Simple and understandable formal derivation of the Hermite is crucial for the understanding it. Especially, within computer graphics and geometric modeling courses, mostly only the mathematical definition is presented. The standard derivation of the Hermite form is quite complex.

The presented approach based on tensor product with linear operators is simple, easy to understand especially convenient for computer graphics and geometric modeling introductory courses. The Bézier parametric patch $S(u, v)$, Bézier[3], is actually based on the tensor product of Bézier curves, i.e. $S(u, v) = C(\mathbf{u}) \otimes C(\mathbf{v})$. In general, cubic parametric curves and bicubic parametric patches are described in Cogen[4], Goldman[6], Prautzsch[11], Holliday[7] and Rockwood[12], etc.

It should be noted, that in the case of the bicubic parametric patches the "border curves are cubic parametric curves. However, the diagonal and anti-diagonal curves, i.e. for $u = v$ and $u = 1 - v$, the curves are of degree 6. If restrictions to the curve degree 3 are introduced, additional requirements are obtained. Such restrictions were specified in Skala[14][16] for the Hermite parametric patch and Kolcun[9], Skala[15] for the Bézier parametric patch. Geometric

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interpretation of the tensor-product diagonal of a Bézier volume was investigated by Holliday and Farin[7]. Triangular patches were desrcibed in Farin[5], Karim[8].

2 Tensor product

The tensor product[19] is not frequently used, however it is very useful. Generally, it is the non-commutative product on two vectors $\mathbf{v} = [v_1, v_2, \ldots, v_n]^T$ and $\mathbf{w} = [w_1, w_2, \ldots, w_m]^T$ defined as:

$$\mathbf{v} \otimes \mathbf{w} = \begin{bmatrix} v_1 w_1 & v_1 w_2 & \cdots & v_1 w_m \\ v_2 w_1 & v_2 w_2 & \cdots & v_2 w_m \\ \vdots & \vdots & \ddots & \vdots \\ v_n w_1 & v_n w_2 & \ldots & v_n w_m \end{bmatrix} \tag{1}$$

The Kronecker product[17], named after the German mathematician Leopold Kronecker (1823–1891), is a generalization of the outer product and a special case of the tensor product[19].

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} = \\ \begin{bmatrix} a_{1,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{1,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \\ a_{2,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{2,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \end{bmatrix} = \\ \begin{bmatrix} a_{1,1}b_{1,1} & a_{1,1}b_{1,2} & a_{1,2}b_{1,1} & a_{1,2}b_{1,2} \\ a_{1,1}b_{2,1} & a_{1,1}b_{2,2} & a_{1,2}b_{2,1} & a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} & a_{2,1}b_{1,2} & a_{2,2}b_{1,1} & a_{2,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,1}b_{2,2} & a_{2,2}b_{2,1} & a_{2,2}b_{2,2} \end{bmatrix} \tag{2}$$

If the tensor product is applied on differential operators, the following matrix is obtained:

$$\begin{bmatrix} 1 \\ \frac{\partial}{\partial u} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \frac{\partial}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\partial}{\partial v} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial u}(\frac{\partial}{\partial v}) \end{bmatrix} = \begin{bmatrix} 1 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial u} & \frac{\partial^2}{\partial u \partial v} \end{bmatrix} \tag{3}$$

The tensor and Kronecker products are multilinear[18] and can be also applied on functions Mochizuki[10].

3 Hermite cubic curve

The Hermite parametric cubic curve segment uses two end-points x_1, x_2 and two tangential vectors x_3, x_4 of the cubic segment end-points, see Fig.1.
The position of the point $x(u)$ is given by Eq.4:

$$x(u) = a_1 u^3 + a_2 u^2 + a_3 u + a_4 \\ x(u) = \sum_{i=1}^{4} a_i u^{4-i} \quad , \quad u \in <0, 1> \tag{4}$$

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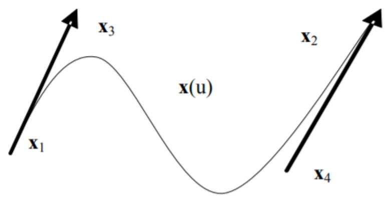

Fig. 1: Hermite cubic curve (tangential vectors shortened

and the tangent vector $x'(u) = \frac{dx(u)}{du}$ is given by Eq.5:

$$x^{(u)}(u) = 3a_1u^2 + 2a_2u + a_3$$
$$x^{(u)}(u) = \sum_{i=1}^{3}(4-i)\ a_i u^{3-i}, \quad u \in <0,1> \tag{5}$$

The Eq.4 can be rewritten using the *dot product* as

$$x(u) = [a_1, a_2, a_3, a_4]^T\ [u^3, u^2, u, 1] = \mathbf{a}^T\mathbf{u} \tag{6}$$

Solving the Eq.4 and Eq.5 for the curve segment end-points, i.e. $u = 0$ and $u = 1$ the following system of linear equations is obtained:

$$\begin{aligned} x(0) &= a_4 \\ x(1) &= a_1 + a_2 + a_3 + a_4 \\ x^{(u)}(0) &= a_3 \\ x^{(u)}(1) &= 3a_1 + 2a_2 + a_3 \end{aligned} \tag{7}$$

where $x^{(u)} = \frac{\partial x}{\partial u}$.
It leads to a system of equations for the unknown coefficients $\mathbf{a} = [a_1, a_2, a_3, a_4]^T$ for the given end-points property
$\boldsymbol{\xi} = [x(0), x(1), x^{(u)}(0), x^{(u)}(1)]^T \stackrel{\text{def}}{=} [x_1, x_2, x_1^{(u)}, x_2^{(u)}]^T$.

$$\begin{bmatrix} 0\,0\,0\,1 \\ 1\,1\,1\,1 \\ 0\,0\,1\,0 \\ 3\,2\,1\,0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_1^{(u)} \\ x_2^{(u)} \end{bmatrix} \quad , \quad \mathbf{Ba} = \boldsymbol{\xi} \tag{8}$$

Solving the linear system of equations $\mathbf{Ba} = \boldsymbol{\xi}$, Eq.7, the coefficients of the Hermite form are obtained.
Then

$$\begin{aligned} x(u) = a_1u^3 + a_2u^2 + a_3u + a_4 = \\ (\mathbf{B}^{-1}\boldsymbol{\xi})^T\mathbf{u} = \boldsymbol{\xi}^T\mathbf{B}^{-T}\mathbf{u} \end{aligned} \tag{9}$$

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where $\mathbf{u} = [u^3, u^2, u, 1]^T$, $\mathbf{B}^{-T}$ is transposed the inverse matrix. Now, the Hermite parametric curve is then described as:

$$x(u) = \boldsymbol{\xi}^T \mathbf{M}_H \mathbf{u} = \boldsymbol{\xi}^T \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \mathbf{u} = \mathbf{u}^T \mathbf{M}_H^T \boldsymbol{\xi} \tag{10}$$

where $\mathbf{u} = [u^3, u^2, u, 1]^T$, $\mathbf{M}_H = \mathbf{B}^{-1}$ is the matrix of the Hermite form and $\boldsymbol{\xi} = [x(0), x(1), x^{(u)}(0), x^{(u)}(1)]^T \equiv [x_1, x_2, x_1^{(u)}, x_2^{(u)}]^T$ are the control values of the curve $x(u)$.

It should be noted that the Eq.10 represents only the $x(u)$-coordinate and for the other coordinates, i.e. $y(u)$, $z(u)$, it is similar.

Generally for the E^3 case, for a curve $\mathbf{C}(\mathbf{u})$ we can write:

$$\mathbf{C}(u) = [\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4]^T \; \mathbf{M}_H \; [u^3, u^2, u, 1] = [\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4]^T \; \mathbf{M}_H \; \mathbf{u} \tag{11}$$

where $\mathbf{P}_1 = [x_1, y_1, z_1]^T$, $\mathbf{P}_2 = [x_2, y_2, z_2]^T$ are vectors of the curve end-points, $\mathbf{P}_3 = [x_1^{(u)}, y_1^{(u)}, z_1^{(u)}]^T$, $\mathbf{P}_4 = [x_2^{(u)}, y_2^{(u)}, z_2^{(u)}]^T$ are vectors of the tangential vectors at the curve end-points. The Eq.10 can be rewritten as:

$$\mathbf{C}(u) = [\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4]^T \; \mathbf{M}_H \; \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix} \tag{12}$$

, i.e.

$$\begin{bmatrix} x(u) \\ y(u) \\ z(u) \end{bmatrix} = \begin{bmatrix} x_1^{(u)} & x_2^{(u)} & x_3^{(u)} & x_4^{(u)} \\ y_1^{(u)} & y_2^{(u)} & y_3^{(u)} & y_4^{(u)} \\ z_1^{(u)} & z_2^{(u)} & z_3^{(u)} & z_4^{(u)} \end{bmatrix} \; \mathbf{M}_H \; \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix} \tag{13}$$

Note that the Eq.10 is formally valid also for the Bézier, Catmul, Ferguson, etc. curves, however, the control vector $\boldsymbol{\xi}$ has different properties.

The Bézier curve of the degree n is defined as:

$$^{(B)}x(u) = \sum_{i=0}^{n} x_i \binom{n}{i} u^i (1-u)^{n-i} \tag{14}$$

and the tangential vectors are defined as:

$$x^{(u)}(0) = n(x_1 - x_0) \quad x^{(u)}(1) = n(x_n - x_{n-1}) \tag{15}$$

It can be seen direct connection between the Hermite and Bézier forms. Therefore the Hermite, Bézier, Ferguson, etc. curves are mutually convertible, see Anand[2].

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It should be noted, that an invertible matrix $\mathbf{M}_{H\rightarrow B}$ (4×4) exists, which transforms the Hermite form to the Bézier form:

$$^{(B)}x(u) = \mathbf{M}_{H\rightarrow B}\ ^{(H)}x(u) \tag{16}$$

where $^{(B)}x(u)$, resp. $^{(H)}x(u)$ means the x-coordinate of the Hermite cubic curve, resp. Bézier cubic curve, see Anand[2]. Continuity conditions were also studied in Ali[1], Skala[13].

4 Hermite patch

The Hermite bicubic patch is actually two dimensional case of the multi-variate interpolation. The bicubic parametric patch for the x-coordinate is defined as:

$$\begin{aligned} x(u,v) = (\sum_{i=1}^{4} a_i u^{4-i})(\sum_{j=1}^{4} b_j v^{4-j}) = \\ \sum_{i=1}^{4}\sum_{j=1}^{4} a_i b_j\ u^{4-i} v^{4-j} = \\ \sum_{i=1}^{4}\sum_{j=1}^{4} u^{4-i}\ s_{i,j}\ v^{4-j} \end{aligned} \tag{17}$$

It can be seen that the Eq.17 can be rewritten as:

$$x(u,v) = \mathbf{u}^T \mathbf{S}\ \mathbf{v} \tag{18}$$

where: $\mathbf{u} = [u^3, u^2, u, 1]^T$, $\mathbf{v} = [v^3, v^2, v, 1]^T$ and the matrix $\mathbf{S}$ has the $s_{i,j}$ elements. Similarly for the y and z-coordinates.

Using the tensor product on functions a simple formula is obtained:

$$x(u,v) = x(u) \otimes x(v) \tag{19}$$

The Eq.18 describes a parametric patch $x(u,v)$. Each point of the curve $x(u)$ in the Eq.10 is parameterized by the second parameter v. As the

$$x(u,v) = \mathbf{u}^T \mathbf{M}_H^T \boldsymbol{\xi}(\mathbf{v}) \tag{20}$$

where: $\boldsymbol{\xi}(\mathbf{v}) = [x_1(v), x_2(v), x_1^{(v)}(u), x_2^{(v)}(u)]^T$ It should be noted that all elements of the vector $\boldsymbol{\xi}(\mathbf{v})$ are the Hermite curves again. It means, that

$$\begin{aligned} x_1(v) = [x_{11}, x_{12}, x_{11}^{(v)}, x_{12}^{(v)}]\mathbf{M}_H \mathbf{v} \\ x_2(v) = [x_{21}, x_{22}, x_{21}^{(v)}, x_{22}^{(v)}]\mathbf{M}_H \mathbf{v} \\ x_1^{(v)}(u) = [x_{11}^{(u)}, x_{12}^{(u)}, x_{11}^{(uv)}, x_{12}^{(uv)}]\mathbf{M}_H \mathbf{v} \\ x_2^{(v)}(u) = [x_{21}^{(u)}, x_{22}^{(u)}, x_{21}^{(uv)}, x_{22}^{(uv)}]\mathbf{M}_H \mathbf{v} \end{aligned} \tag{21}$$

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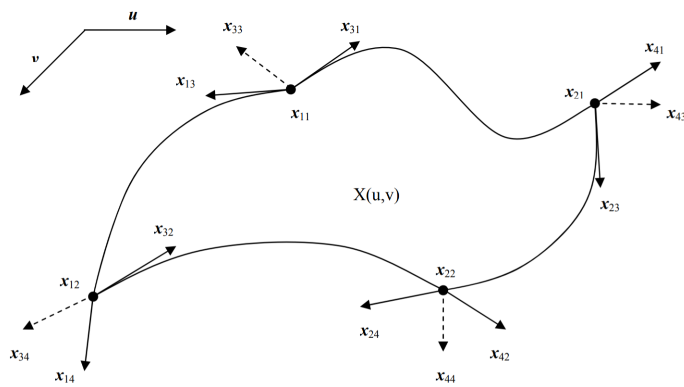

Fig. 2: Hermite bi-cubic patch (tangential and twist vectors scaled)

where $x^{(uv)} \overset{\text{def}}{=} \frac{\partial^2 x}{\partial u \partial v}$ and $\mathbf{v} = [v^3, v^2, v, 1]^T$.

Now, the Hermite patch $x(u, v)$ for the x-coordinate can be rewritten as:

$$x(u,v) = \mathbf{u}^T \mathbf{M}_H^T \begin{bmatrix} x_{11} & x_{12} & x_{11}^{(v)} & x_{12}^{(v)} \\ x_{21} & x_{22} & x_{21}^{(v)} & x_{22}^{(v)} \\ x_{11}^{(u)} & x_{12}^{(u)} & x_{11}^{(uv)} & x_{12}^{(uv)} \\ x_{21}^{(u)} & x_{22}^{(u)} & x_{21}^{(uv)} & x_{22}^{(uv)} \end{bmatrix} \mathbf{M}_H \mathbf{v} \tag{22}$$

or using more compact form:

$$x(u,v) = \mathbf{u}^T \mathbf{M}_H^T \; \mathbf{X} \; \mathbf{M}_H \mathbf{v} \tag{23}$$

where the matrix $\mathbf{X}$ is the matrix of the control values of the Hermite patch form.

It can be seen, that it is the bi-quadratic form. This formal notation is common for the other bicubic patches, e.g. for the Bézier, Ferguson, etc.

5 Hermite bicubic plate using tensor product

The Hermite bicubic plate can be expressed by using tensor product as:

$$\mathbf{S}(u,v) = \mathbf{C}(u) \otimes \mathbf{C}(v) \tag{24}$$

Using the tensor product and more compact form with the block matrix notation:

$$x(u,v) = \mathbf{u}^T \mathbf{M}_H^T \begin{bmatrix} x_{11} & x_{12} & x_{11}^{(v)} & x_{12}^{(v)} \\ x_{21} & x_{22} & x_{21}^{(v)} & x_{22}^{(v)} \\ x_{11}^{(u)} & x_{12}^{(u)} & x_{11}^{(uv)} & x_{12}^{(uv)} \\ x_{21}^{(u)} & x_{22}^{(u)} & x_{21}^{(uv)} & x_{22}^{(uv)} \end{bmatrix} \mathbf{M}_H \mathbf{v} \tag{25}$$

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The Eq.25 can be rewritten to:

$$x(u,v) = \mathbf{u}^T \mathbf{M}_H^T \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} \mathbf{M}_H \mathbf{v} \tag{26}$$

where x_{ij} are the control values, see Fig.2.

Now, using the tensor product and a submatrix 4×4 of the patch end-points a more "compact form" describing the Hermite patch is obtained as:

$$\mathbf{P}(u,v) = \mathbf{u}^T \mathbf{M}_H^T \begin{bmatrix} 1 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial u} & \frac{\partial^2}{\partial u \partial v} \end{bmatrix} \otimes \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \mathbf{M}_H \mathbf{v} =$$

$$\mathbf{u}^T \mathbf{M}_H^T \begin{bmatrix} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} & \frac{\partial}{\partial v} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \\ \frac{\partial}{\partial u} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} & \frac{\partial^2}{\partial u \partial v} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \end{bmatrix} \mathbf{M}_H \mathbf{v} \tag{27}$$

If the differential tensor operator Eq.3 is applied on the Hermit bicubic corners, the matrix form is obtained as follows:

$$\begin{gathered} \begin{bmatrix} 1 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial u} & \frac{\partial^2}{\partial u \partial v} \end{bmatrix} \otimes \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} = \\ \begin{bmatrix} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} & \frac{\partial}{\partial v} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \\ \frac{\partial}{\partial u} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} & \frac{\partial^2}{\partial u \partial v} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \end{bmatrix} = \\ \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \frac{\partial}{\partial u} \mathbf{P}_{11} & \frac{\partial}{\partial u} \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} & \frac{\partial}{\partial y} \mathbf{P}_{21} & \frac{\partial}{\partial y} \mathbf{P}_{22} \\ \frac{\partial}{\partial v} \mathbf{P}_{11} & \frac{\partial}{\partial v} \mathbf{P}_{12} & \frac{\partial^2}{\partial u \partial v} \mathbf{P}_{11} & \frac{\partial^2}{\partial u \partial v} \mathbf{P}_{12} \\ \frac{\partial}{\partial v} \mathbf{P}_{21} & \frac{\partial}{\partial v} \mathbf{P}_{22} & \frac{\partial^2}{\partial u \partial v} \mathbf{P}_{21} & \frac{\partial^2}{\partial u \partial v} \mathbf{P}_{22} \end{bmatrix} \end{gathered} \tag{28}$$

It can be seen that this matrix clearly shows the Hermite form properties.

It should be noted that the Hermite bicubic parametric patch can be converted to the Bézier bicubic patch similarly as in the case of cubic curves, see Anand[2].

6 Conclusion

This contribution presents a different approach for the Hermite cubic curve and Hermite bicubic patch definition using the tensor product, as the tensor product might be applied not only on vectors and matrices, but also on functions. Unfortunately, especially in computer graphics courses, vector or matrix formulation is presented only, without deeper derivation of the formulas.

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The derivation of the Hermite form using the tensor matrix operations is more understandable, especially in the case of the Hermite bicubic patch.

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