Polynomial multiplication C(x)=A(x)*B(x) using tables: An alternative tool for teaching mathematics

1. Introduction

The polynomial multiplication, if made by the "item by item" approach Wiki (2023a), combines two steps:

- multiplication of the polynomial coefficients, and concurrently
- summation of the polynomial item's exponents.

These two simultaneous steps lead to errors and difficulties in understanding. Polynomial multiplication is actually a tensor product Wiki (2023b) applied on two polynomials represented by Tab.1. It should be noted, that in some courses, the polynomials are handled as a purely abstract algebraic structure and students do not have a geometric interpretation.

2. Polynomial multiplication

Let us consider, for a sake of simplicity, two polynomials $A(x)$ of the degree n_A and $B(x)$ of the degree n_A of one variable. Then the polynomial $C(x)$ is given as $C(x) = A(x) * B(x)$ of the degree $n_A + n_B$.
Let us consider two polynomials[1] :

$$A(x) = 3x^0 + 1x^1 + 4x^2 + 7x^3 + 6x^4 \quad , \quad B(x) = 2x^0 + 3x^1 + 1x^2 + 5x^2$$

Then the polynomial given as $C(x) = A(x) * B(x)$ can be expressed by table Tab.1:

Table 1. Multiplication of $A(x)$ and $B(x)$ polynomials

	exponent	0	1	2	3	4
exponent	coefficient	3	1	4	7	6
0	2	6	2	8	14	12
1	3	9	3	12	21	18
2	1	3	1	4	7	6
3	5	15	5	20	35	30

Using the table interpretation leads numerically to two simple consecutive operations, as only two coefficient values are multiplied, i.e. $coefficient_i * coefficient_j$ for each $exponent_i$ and $exponent_j$. Summing the values on the anti-diagonals in the Tab.1, i.e. $exponent_i + exponent_j = const$, coefficients for each exponent of the polynomial $C(x)$ are obtained.

Table 2. Coefficients for a constant exponent

exponent	0	1	2	3	4	5	6	7
coefficient	6	11	14	42	42	45	41	30

[1] It is necessary to explain to students that $x^0 = 1, x^1 = x$.

The resulting polynomial $C(x) = A(x) * B(x)$ is of the degree 7, see Tab.2.

The resulting polynomial $C(x)$ is given by Eq.1, see Tab.2:

$$C(x) = 6x^0 + 11x^1 + 14x^2 + 42x^3 + 42x^4 + 45x^6 + 41x^6 + 30x^7 \tag{1}$$

This polynomial multiplication method splits the polynomial multiplication into two consecutive steps. It is resulting in a better and faster understanding of the polynomial multiplication computation. clearpage The presented approach can also be applied for polynomials with multidimensional arguments, i.e. $C(\mathbf{x}) = A(\mathbf{x}) * B(\mathbf{x})$. Let us consider two polynomials $A(\mathbf{x}) = (x + 2y)$ and $B(\mathbf{x}) = (3x - 4y + 5)$. Then the resulting polynomial $C(\mathbf{x})$ is given by Eq.2:

$$\begin{aligned}(x + 2y)(3x - 4y + 5) &= 3x^2 - 4xy + 5x + 6xy - 8y^2 + 10y \\ &= 3x^2 + 2xy + 5x - 8y^2 + 10y\end{aligned} \tag{2}$$

In this case, the table items must also contain variables, see Tab.3.

As the multiplication of items has no constant exponents, i.e. there is no order in exponents, the summation of elements in Tab.3 is a bit complicated. As a result, we obtain:

Table 3. Multiplication of $A(\mathbf{x}) * B(\mathbf{x})$ polynomials

items	$3x$	$-4y$	5
x	$3x^2$	$-4xy$	$5x$
$2y$	$6xy$	$-8y^2$	$10y$

$$(x + 2y)(3x - 4y + 5) = 3x^2 + 2xy + 5x - 8y^2 + 10y \tag{3}$$

It can be seen that this approach enables solving even complicated cases more reliably and limit errors in students' course-works.

3. Conclusion

A simple methodology for polynomial multiplication using a table representation has been presented. It uses a tensor product represented by a table, which reduces mistakes caused by omitting some mutual polynomial items in the multiplication of $A(x)$ and $B(x)$ polynomials. It is also applicable in multidimensional cases. A similar approach can be used for the polynomial summation, too.

This presented approach was used in classes with pupils at a grammar school with positive results.

There is a hope that this contribution can help teachers in more efficient teaching this topic.

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